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Information Technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C — Part 1: Binary floating-point arithmetic

Technologies de l'information — Langages de programmation, leurs environnements et interfaces du logiciel système — Extensions à virgule flottante pour C — Partie I: Binary arithmétique flottante

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## **Foreword**

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ISO nnn-n was prepared by Technical Committee ISO JTC 1, *Information Technology*, Subcommittee SC 22, *Programming languages, their environments, and system software interfaces*.

ISO nnn consists of the following parts, under the general title Floating-point extensions for C:

- Part 1: Binary floating-point arithmetic
- Part 2: Decimal floating-point arithmetic
- Part 3: Interchange and extended types
- Part 4: Supplemental functions
- Part 5: Supplemental attributes

Part 1 updates ISO/IEC 9899:2011 (Information technology — Programming languages, their environments and system software interfaces — Programming Language C), Annex F in particular, to support all required features of ISO/IEC/IEEE 60559:2008 (Information technology — Microprocessor Systems — Floating-point arithmetic).

Part 2 supersedes ISO/IEC TR 24732:2008 (Information technology – Programming languages, their environments and system software interfaces – Extension for the programming language C to support decimal floating-point arithmetic.

Parts 3-5 specify extensions to ISO/IEC 9899:2011 for features recommended in ISO/IEC/IEEE 60559:2008.

## Introduction

A paragraph.

The **introduction** is an optional preliminary element used, if required, to give specific information or commentary about the technical content of the document, and about the reasons prompting its preparation. It shall not contain requirements.

The introduction shall not be numbered unless there is a need to create numbered subdivisions. In this case, it shall be numbered 0, with subclauses being numbered 0.1, 0.2, etc. Any numbered figure, table, displayed formula or footnote shall be numbered normally beginning with 1.

WORKING DRAFT ISO/WD nnn-n

Information Technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C — Part 1: Binary floating-point arithmetic

## 1 Scope

Part 1 of Technical Specification 00000 extends programming language C to support binary floating-point arithmetic conforming to ISO/IEC/IEEE 60559:2011. It covers all requirements of IEC 60559 as they pertain to C floating types that use IEC 60559 binary formats.

TS 00000-1 does not cover decimal floating-point arithmetic, nor most other optional features of IEC 60559

TS 00000-1 is primarily an update to C11 Annex F. However, it proposes that the new interfaces that are suitable for general implementations be added in the Library (7) clauses of C11. Also it includes a few auxiliary changes in C11 where the specification is problematic for IEC 60559 support.

#### 2 Conformance

An implementation conforms to Part 1 of this Technical Specification if

- a) It meets the requirements for a conforming hosted implementation of C11 with all the suggested changes to C11 in this Technical Specification; and
- b) It defines STDC IEC 60559 to 201ymmL.

ISSUE: Should we define conformance for freestanding implementations too? Note that IEC 60559 requirements for conversions between floating-point formats and decimal character sequences are met in <stdio.h>.

### 3 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/IEC 9899:2011, Information technology — Programming languages, their environments and system software interfaces — Programming Language C

ISO/IEC/IEEE 60559:2011, Information technology — Microprocessor Systems — Floating-point arithmetic

IEEE 754-2008, *IEEE Standard for Floating-Point Arithmetic*. The Institute of Electrical and Electronic Engineers, Inc., New York, 2008

### 4 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC/IEEE 60559:2011 and the following apply.

## 4.1

### C11

standard ISO/IEC 9899:2011, Information technology — Programming languages, their environments and system software interfaces — Programming Language C

#### 5 Predefined macros

The following macro is conditionally defined by the implementation:

\_\_STDC\_IEC\_60559\_1\_\_ The integer constant 201ymmL, intended to indicate conformance to Part 1 of this Technical Specification.

## 6 Revised floating-point standard

C11 Annex F specifies C language support for the floating-point arithmetic of IEC 60559:1989. This document proposes changes to C11 to bring Annex F into alignment with IEC 60559:2011.

### Suggested change to C11:

Change F.1 from:

#### **F.1 Introduction**

This annex specifies C language support for the IEC 60559 floating-point standard. The *IEC 60559 floating-point standard* is specifically *Binary floating-point arithmetic for microprocessor systems, second edition* (IEC 60559:1989), previously designated IEC 559:1989 and as *IEEE Standard for Binary Floating-Point Arithmetic* (ANSI/IEEE 754–1985). *IEEE Standard for Radix-Independent Floating-Point Arithmetic* (ANSI/IEEE 854–1987) generalizes the binary standard to remove dependencies on radix and word length. *IEC 60559* generally refers to the floating-point standard, as in IEC 60559 operation, IEC 60559 format, etc. An implementation that defines **\_\_STDC\_IEC\_559\_\_** shall conform to the specifications in this annex.356) Where a binding between the C language and IEC60559 is indicated, the IEC 60559-specified behavior is adopted by reference, unless stated otherwise. Since negative and positive infinity are representable in IEC 60559 formats, all real numbers lie within the range of representable values.

to:

#### **F.1 Introduction**

This annex specifies C language support for the IEC 60559 floating-point standard. The *IEC* 60559 floating-point standard is specifically *Floating-point arithmetic* (ISO/IEC/IEEE 60559:2011), also designated as *IEEE Standard for Floating-Point Arithmetic* (IEEE 754–2008). The IEC 60559 floating-point standard supersedes the IEC 60559:1989 binary arithmetic standard, also designated as *IEEE Standard for Binary Floating-Point Arithmetic* (IEEE 754–1985). The IEC 60559 floating-point standard specifies decimal, as well as binary, floating-point arithmetic, superseding *IEEE Standard for Radix-Independent Floating-Point Arithmetic* (ANSI/IEEE854–1987), which generalized the binary standard to remove dependencies on radix and word length. *IEC* 60559 generally refers to the floating-point standard, as in IEC 60559 operation, IEC 60559 format, etc. An implementation that defines \_\_STDC\_IEC\_60559\_\_ to 201ymmL shall conform to the specifications in this annex.356) Where a binding between the C language and IEC 60559 is indicated, the IEC 60559-specified behavior is adopted by reference, unless stated otherwise. Since negative and positive infinity are representable in IEC 60559 formats, all real numbers lie within the range of representable values.

In footnote 356), change "\_\_STDC\_IEC\_559\_\_" to "\_\_STDC\_IEC\_60559\_\_".

ISSUE: Are there any compatibility issues for code using \_\_STDC\_IEC\_559\_\_ (6.10.8.3)? We believe not, but are leaving the issue as a caution.

ISSUE: What change is appropriate for \_\_STDC\_IEC\_559\_COMPLEX?

ISSUE: Do we need a WANT macro to guard the new interfaces? Should each part of the TS have its own WANT macro?

### 7 Types

## 7.1 Terminology

IEC 60559 now includes a 128-bit binary format as one of its three binary basic formats: binary32, binary64, and binary128. The binary128 format continues to meet the less specific requirements for a binary64-extended format, as in the previous IEC 60559. The suggested changes to C11 below reflect the new terminology in IEC 60559; these changes are not substantive.

## Suggested changes to C11:

In F.2, change the third bullet from:

The long double type matches an IEC 60559 extended format, 357) else a non-IEC 60559 extended format, else the IEC 60559 double format.

to:

 The long double type matches the IEC 60559 binary128 format, else an IEC 60559 binary64extended format,357) else a non-IEC 60559 extended format, else the IEC 60559 binary64 format.

In F.2, change the sentence after the bullet from:

Any non-IEC 60559 extended format used for the **long double** type shall have more precision than IEC 60559 double and at least the range of IEC 60559 double.358)

to:

Any non-IEC 60559 extended format used for the **long double** type shall have more precision than IEC 60559 binary64 and at least the range of IEC 60559 binary64.358)

Change footnote 357) from:

357) "Extended" is IEC 60559's double-extended data format. Extended refers to both the common 80-bit and quadruple 128-bit IEC 60559 formats.

to:

357) IEC 60559 binary64-extended formats include the common 80-bit IEC 60559 format.

In F.2, change the recommended practice from:

### Recommended practice

The **long double** type should match an IEC 60559 extended format.

to:

#### Recommended practice

The **long double** type should match the IEC 60559 binary128 format, else an IEC 60559 binary64-extended format.

## 7.2 Canonical encodings

IEC 60559 refers to preferred encodings in a format as *canonical*. Some formats also contain redundant or ill-specified encodings, which are non-canonical. All encodings In IEC 60559 binary interchange formats are canonical; however, its extended formats may have non-canonical encodings. (IEC 60559 decimal interchange formats, covered in Part 2 of this Technical Specification, contain non-canonical encodings.)

## Suggested change to C11:

After 5.2.4.2.2 #5, add:

The preferred encodings in a floating type are called *canonical*. A floating type may also contain *non-canonical* encodings, for example, redundant encodings of some or all of its values, or encodings that are extraneous to the floating-point model. Typically, floating-point operations deliver results with canonical encodings.

## 8 Operation binding

IEC 60559 includes several new required operations. Table 1 in the suggested change to C11 below shows the complete mapping of IEC 60559 operations to C operators, functions, and function-like macros. The new IEC 60559 operations map to C functions and function-like macros; no new C operators are proposed.

### Suggested change to C11:

Replace F.3:

### F.3 Operators and functions

C operators and functions provide IEC 60559 required and recommended facilities as listed below.

- The +, -, \*, and / operators provide the IEC 60559 add, subtract, multiply, and divide operations.
- The **sqrt** functions in **<math.h>** provide the IEC 60559 square root operation.
- The **remainder** functions in **<math.h>** provide the IEC 60559 remainder operation. The **remquo** functions in **<math.h>** provide the same operation but with additional information.
- The rint functions in <math.h> provide the IEC 60559 operation that rounds a floating-point number to an integer value (in the same precision). The nearbyint functions in <math.h> provide the nearbyinteger function recommended in the Appendix to ANSI/IEEE 854.
- The conversions for floating types provide the IEC 60559 conversions between floating-point precisions.
- The conversions from integer to floating types provide the IEC 60559 conversions from integer to floating point.
- The conversions from floating to integer types provide IEC 60559-like conversions but always round toward zero.
- The Irint and IIrint functions in <math.h> provide the IEC 60559 conversions, which honor the directed rounding mode, from floating point to the long int and long long int integer formats. The Irint and IIrint functions can be used to implement IEC 60559 conversions from floating to other integer formats.
- The translation time conversion of floating constants and the strtod, strtof, strtold, fprintf, fscanf, and related library functions in <stdlib.h>, 359) Since NaNs created by IEC 60559 operations are always quiet, quiet NaNs (along with infinities) are sufficient for closure of the arithmetic.

- <stdio.h>, and <wchar.h> provide IEC 60559 binary-decimal conversions. The strtold function in <stdlib.h> provides the conv function recommended in the Appendix to ANSI/IEEE 854.
- The relational and equality operators provide IEC 60559 comparisons. IEC 60559 identifies a need for additional comparison predicates to facilitate writing code that accounts for NaNs. The comparison macros (isgreater, isgreaterequal, isless, islessequal, islessgreater, and isunordered) in <math.h> supplement the language operators to address this need. The islessgreater and isunordered macros provide respectively a quiet version of the <> predicate and the unordered predicate recommended in the Appendix to IEC 60559.
- The feclearexcept, feraiseexcept, and fetestexcept functions in <fenv.h> provide the facility to test and alter the IEC 60559 floating-point exception status flags. The fegetexceptflag and fesetexceptflag functions in <fenv.h> provide the facility to save and restore all five status flags at one time. These functions are used in conjunction with the type fexcept\_t and the floating-point exception macros (FE\_INEXACT, FE\_DIVBYZERO, FE\_UNDERFLOW, FE\_OVERFLOW, FE\_INVALID) also in <fenv.h>.
- The fegetround and fesetround functions in <fenv.h> provide the facility to select among the IEC 60559 directed rounding modes represented by the rounding direction macros in <fenv.h> (FE\_TONEAREST, FE\_UPWARD, FE\_DOWNWARD, FE\_TOWARDZERO) and the values 0, 1, 2, and 3 of FLT\_ROUNDS are the IEC 60559 directed rounding modes.
- The **fegetenv**, **feholdexcept**, **fesetenv**, and **feupdateenv** functions in **fenv.h** provide a facility to manage the floating-point environment, comprising the IEC 60559 status flags and control modes.
- The copysign functions in <math.h> provide the copysign function recommended in the Appendix to IEC 60559.
- The **fabs** functions in **<math.h>** provide the abs function recommended in the Appendix to IEC 60559.
- The unary minus (-) operator provides the unary minus (-) operation recommended in the Appendix to IEC 60559.
- The scalbn and scalbln functions in <math.h> provide the scalb function recommended in the Appendix to IEC 60559.
- The **logb** functions in **<math.h>** provide the logb function recommended in the Appendix to IEC 60559, but following the newer specifications in ANSI/IEEE 854.
- The **nextafter** and **nexttoward** functions in **<math.h>** provide the nextafter function recommended in the Appendix to IEC 60559 (but with a minor change to better handle signed zeros).
- The isfinite macro in <math.h> provides the finite function recommended in the Appendix to IEC 60559.
- The isnan macro in <math.h> provides the isnan function recommended in the Appendix to IEC 60559.
- The signbit macro and the fpclassify macro in <math.h>, used in conjunction with the number classification macros (FP\_NAN, FP\_INFINITE, FP\_NORMAL, FP\_SUBNORMAL, FP\_ZERO), provide the facility of the class function recommended in the Appendix to IEC 60559 (except that the classification macros defined in 7.12.3 do not distinguish signaling from quiet NaNs).

with:

### F.3 Operations

C operators, functions, and function-like macros provide the operations required by IEC 60559 as shown in the following table. Specifications for the C facilities are provided in other clauses.

Table 1 — Operation binding

| 150.00550                                | 0 1 15 11 1                    |
|--|--------------------------------|
| IEC 60559 operation                      | C operator/function/macro      |
| roundToIntegralTiesToEven                | roundeven                      |
| roundToIntegralTiesAway                  | round                          |
| roundToIntegralTowardZero                | trunc                          |
| roundToIntegralTowardPositive            | ceil                           |
| roundToIntegralTowardNegative            | floor                          |
| roundToIntegralExact                     | rint                           |
| nextUp                                   | nextup                         |
| nextDown                                 | nextdown                       |
| remainder                                | remainder, remquo              |
| minNum                                   | fmin                           |
| maxNum                                   | fmax                           |
| minNumMag                                | fminmag                        |
| maxNumMag                                | fminmax                        |
| scaleB                                   | scalbn, scalbln                |
| logB                                     | logb, ilogb, llogb             |
| addition                                 | +                              |
| formatOf addition with narrower format   | fadd, faddl, daddl             |
| subtraction                              | -                              |
| formatOf subtraction with narrower       | fsub, fsubl, dsubl             |
| format                                   |                                |
| multiplication                           | *                              |
| formatOf multiplication with narrower    | fmul, fmull, dmull             |
| format                                   |                                |
| division                                 | 1                              |
| formatOf division with narrower format   | fdiv, fdivl, ddivl             |
| squareRoot                               | sqrt                           |
| formatOf squareRoot with narrower        | fsqrt, fsqrtl, dsqrtl          |
| format                                   |                                |
| fusedMultiplyAdd                         | fma                            |
| formatOf fusedMultiplyAdd with           | ffma, ffmal, dfmal             |
| narrower format                          |                                |
| convertFromInt                           | cast and implicit conversion   |
| convertToIntegerTiesToEven               | fromfp, ufromfp                |
| convertToIntegerTowardZero               | fromfp, ufromfp                |
| convertToIntegerTowardPositive           | fromfp, ufromfp                |
| convertToIntegerTowardNegative           | fromfp, ufromfp                |
| convertToIntegerTiesToAway               | fromfp, ufromfp, Iround,       |
|  | Ilround                        |
| convertToIntegerExactTiesToEven          | fromfpx, ufromfpx              |
| convertToIntegerExactTowardZero          | fromfpx, ufromfpx              |
| convertToIntegerExactTowardPositive      | fromfpx, ufromfpx              |
| convertToIntegerExactTowardNegative      | fromfpx, ufromfpx              |
| convertToIntegerExactTiesToAway          | fromfpx, ufromfpx              |
| convertFormat - different floating types | cast and implicit conversions  |
| convertFormat - same floating type       | canonicalize                   |
| convertFromDecimalCharacter              | strtod, wcstod scanf decimal   |
|  | floating constants             |
| convertToDecimalCharacter                | printf                         |
| convertFromHexCharacter                  | strtod, wcstod, scanf,         |
| - · · · · · · · · · · · · · · · · · · ·  | hexadecimal floating constants |
| convertToHexCharacter                    | printf                         |
| сору                                     | memcpy, memmove                |
| negate                                   | -(x)                           |
| abs                                      | fabs                           |
| copySign                                 | copysign                       |

| compareQuietEqual                                     | ==                                    |
|---|---------------------------------------|
| compareQuietNotEqual                                  | !=                                    |
| compareSignalingEqual                                 | iseqsig                               |
| compareSignalingGreater                               | >                                     |
| compareSignalingGreaterEqual                          | >=                                    |
| compareSignalingLess                                  | <                                     |
| compareSignalingLessEqual                             | <=                                    |
| compareSignalingNotEqual                              | ! iseqsig(x)                          |
| compareSignalingNotGreater                            | ! (x > y)                             |
| compareSignalingLessUnordered                         | ! (x >= y)                            |
| compareSignalingNotLess                               | ! (x < y)                             |
| compareSignalingGreaterUnordered                      | ! (x <= y)                            |
| compareQuietGreater                                   | isgreater                             |
| compareQuietGreaterEqual                              | isgreaterequal                        |
| compareQuietLess                                      | isless                                |
| compareQuietLessEqual                                 | islessequal                           |
| compareQuietUnordered                                 | isunordered                           |
| compareQuietNotGreater                                | ! Isgreater(x, y)                     |
| compareQuietLessUnordered                             | ! Isgreaterequal(x, y)                |
| compareQuietNotLess                                   | ! Isless(x, y)                        |
| compareQuietGreaterUnordered                          | ! Islessequal(x, y)                   |
| compareQuietOrdered                                   | ! Isunordered(x, y)                   |
| class   | fpclassify, signbit,                  |
| Cidoo   | issignaling                           |
| isSignMinus   | signbit                               |
| isNormal  | isnormal                              |
| isFinite  | isfinite                              |
| isZero  | x == 0.0                              |
| isSubnormal   | issubnormal                           |
| isInfinite  | isinf                                 |
| isNaN   | isnan                                 |
| isSignaling   | issignaling                           |
| isCanonical   | iscanonical                           |
| radix   | FLT RADIX                             |
| totalOrder  | totalorder                            |
| totalOrderMag   | totalordermag                         |
| lowerFlags  | feclearexcept                         |
| raiseFlags  | fesetexcept                           |
| testFlags   | fetestexcept                          |
| testSavedFlags  | fetestexceptflag                      |
| restoreFlags  | fesetexceptflag                       |
| saveAllFlags  | fegetexceptflag                       |
|   |                                       |
| getBinaryRoundingDirection                            |                                       |
| getBinaryRoundingDirection setBinaryRoundingDirection | fegetround                            |
| setBinaryRoundingDirection                            | fegetround fesetround                 |
| setBinaryRoundingDirection saveModes                  | fegetround<br>fesetround<br>fesetmode |
| setBinaryRoundingDirection                            | fegetround fesetround                 |

## ISSUE: How to improve the table?

Whether C assignment (and conversion as if by assignment) to the same format is an IEC 60559 convertFormat or copy operation is implementation-defined, even if <fenv.h> defines the macro FE\_SNANS\_ALWAYS\_SIGNAL.

The C **nearbyint** functions provide the nearbyinteger function recommended in the Appendix to ANSI/IEEE 854.

The C **nextafter** and **nexttoward** functions provide the nextafter function recommended in the Appendix to IEC 60559:1989 (but with a minor change to better handle signed zeros).

The C **getpayload**, **setpayload**, and **setpayloadsig** functions provide program access to NaN payloads, defined in IEC 60559.

The C **fegetenv**, **feholdexcept**, **fesetenv**, and **feupdateenv** functions provide a facility to manage the dynamic floating-point environment, comprising the IEC 60559 status flags and dynamic control modes.

## 9 Floating to integer conversion

IEC 60559 allows but does not require floating to integer type conversions to raise the "inexact" floating-point exception for non-integer inputs within the range of the integer type. It recommends that implicit conversions raise "inexact" in these cases.

### Suggested change to C11:

Replace footnote 360):

360) ANSI/IEEE 854, but not IEC 60559 (ANSI/IEEE 754), directly specifies that floating-to-integer conversions raise the "inexact" floating-point exception for non-integer in-range values. In those cases where it matters, library functions can be used to effect such conversions with or without raising the "inexact" floating-point exception. See **rint**, **Irint**, **Ilrint**, and **nearbyint** in **<math.h>**.

with:

360) IEC 60559 recommends that implicit floating-to-integer conversions raise the "inexact" floating-point exception for non-integer in-range values. In those cases where it matters, library functions can be used to effect such conversions with or without raising the "inexact" floating-point exception. See **fromfp**, **ufromfpx**, **ufromfpx**,

## 10 Conversions between floating types and decimal character sequences

IEC 60559 now requires correct rounding for conversions between its supported formats and decimal character sequences with up to *H* decimal digits where

```
H \ge M + 3,

M = 1 + \text{ceiling}(p \times \log_{10}(2))
```

p is the precision of the widest support IEC 60559 binary format.

*M* is large enough that conversion from the widest supported format to a decimal character sequence with *M* decimal digits and back will be the identity function. IEC 60559 also now completely specifies conversions involving more than *H* decimal digits. The following suggested changes to C11 satisfy these requirements.

### Suggested change to C11:

Rename F.5 from:

#### F.5 Binary-decimal conversion

to:

### F.5 Conversions between binary floating types and decimal character sequences

Insert after F.5 #2:

The <float.h> header defines the macro

### CR\_DECIMAL\_DIG

which expands to an integral constant expression suitable for use in **#if** preprocessing directives whose value is a number such that conversions between all supported IEC 60559 binary types and character sequences with at most **CR\_DECIMAL\_DIG** significant decimal digits are correctly rounded. The value of **CR\_DECIMAL\_DIG** shall be at least **DECIMAL\_DIG** + 3. If the implementation correctly rounds for all numbers of significant decimal digits, then **CR\_DECIMAL\_DIG** shall have the value of the macro **UINTMAX\_MAX**.

Conversions of IEC 60559 binary-floating types to character sequences with more than **CR\_DECIMAL\_DIG** significant decimal digits shall correctly round to **CR\_DECIMAL\_DIG** significant digits and pad zeros on the right.

Conversions from character sequences with more than **CR\_DECIMAL\_DIG** significant decimal digits to IEC 60559 binary floating types shall correctly round to an intermediate character sequence with **CR\_DECIMAL\_DIG** significant decimal digits, according to the applicable rounding direction, and correctly round the intermediate result (having **CR\_DECIMAL\_DIG** significant decimal digits) to the destination type. The "inexact" floating-point exception is raised (once) if either conversion is inexact<sup>1</sup>. (The second conversion may raise the "overflow" or "underflow" floating-point exception.)

### 11 Constant rounding directions

IEC 60559 now requires a means for programs to specify constant values for the rounding direction mode for all standard operations in static parts of code (as specified by the programming language). The following suggested changes meet this requirement by adding standard pragmas for specifying constant values for the rounding direction mode. Changes to the C11 specification for the dynamic rounding direction modes in the floating-point environment are needed to accommodate the new pragmas; current programs are not affected by these changes.

#### Suggested changes to C11:

Modify 7.6.1 "The FENV ACCESS pragma" by replacing:

If part of a program tests floating-point status flags, sets floating-point control modes, or runs under non-default mode settings, but was translated with the state for the **FENV\_ACCESS** pragma "off", the behavior is undefined.

with:

If part of a program tests floating-point status flags, sets floating-point control modes, or establishes non-default mode settings using any means other than the **FENV\_ROUND** pragmas, but was translated with the state for the **FENV\_ACCESS** pragma "off", the behavior is undefined.

ISSUE: In order to distinguish the new constant specification of modes (in this TS) from the dynamic specification of modes (already in C11), we plan to prefix "floating-point control mode" with "constant" or "dynamic" and prefix "floating-point environment" with "dynamic", where needed, and provide explanatory definitions and terminology conventions. These changes are not complete in this draft.

Modify footnote 213) by replacing:

In general, if the state of **FENV\_ACCESS** is "off", the translator can assume that default modes are in effect and the flags are not tested.

<sup>&</sup>lt;sup>1</sup> The intermediate conversion is exact only if all input digits after the first **CR\_DECIMAL\_DIG** digits are 0.

with:

In general, if the state of **FENV\_ACCESS** is "off", the translator can assume that the flags are not tested, and that default modes are in effect, except where specified otherwise by an **FENV\_ROUND** pragma.

Following 7.6.1 "The FENV ACCESS pragma", insert:

### 7.6.2 Rounding control pragmas

#### Synopsis

#include <fenv.h>
#pragma STDC FENV\_ROUND direction

#### Description

The **FENV\_ROUND** pragma provides a means to specify a constant rounding direction for binary floating-point operations within a translation unit or compound statement. The pragma shall occur either outside external declarations or preceding all explicit declarations and statements inside a compound statement. When outside external declarations, the pragma takes effect from its occurrence until another **FENV\_ROUND** pragma is encountered, or until the end of the translation unit. When inside a compound statement, the pragma takes effect from its occurrence until another **FENV\_ROUND** pragma is encountered (including within a nested compound statement), or until the end of the compound statement; at the end of a compound statement the static rounding mode is restored to its condition just before the compound statement. If this pragma is used in any other context, its behavior is undefined.

direction shall be one of the rounding direction macro names defined in 7.6, or FE\_DYNAMIC. If any other value is specified, the behavior is undefined. If no FENV\_ROUND pragma is in effect, or the specified constant rounding mode is FE\_DYNAMIC, rounding is according to the mode specified by the floating-point environment, which is the rounding mode that was established either at thread creation or by a call to fesetround, fesetenv, or feupdateenv. If the FE\_DYNAMIC mode is specified and FENV\_ACCESS is "off", the translator may assume that the default global mode is in effect.

ISSUE: While 754 doesn't make requirements for flags beyond the global flags in C, it does allow for what might be called "local" flags, instead of, or in addition to, global flags. The motivation behind static rounding direction attributes is that accessing dynamic modes become synchronization points - which inhibits auto parallelization. This problem seems to exist for global flags too. Do we need to add support for local flags?

Within the scope of an **FENV\_ROUND** directive establishing a mode other than **FE\_DYNAMIC**, all floating-point operations, conversions, and calls to functions declared in **<math.h>**, **<complex.h>**, **<stdio.h>**, and **<stdlib.h>** shall be evaluated according to the specified constant rounding mode<sup>2</sup> (i.e., as though no constant mode was specified and the dynamic rounding mode had been established by a call to **fesetround**). Floating constants (6.4.4.2) that occur in the scope of a constant rounding mode are interpreted according to that mode.

Calls to functions other than those indicated above are not affected by constant rounding modes<sup>3</sup> — they see (and affect) only the dynamic mode.

Any call made through a function pointer, even if it is to a function which would otherwise be affected by the constant rounding mode, is not affected by the constant rounding mode. It sees (and affects) only the dynamic rounding mode.

\_

<sup>&</sup>lt;sup>2</sup> Note that the **<fenv.h>** functions are not included in this list.

<sup>&</sup>lt;sup>3</sup> Thus, an implementation on hardware that supports only global floating-point rounding control must restore the mode that was in effect on entry to the block governed by the constant rounding mode before calling any such function, unless it can prove that the function being called is unaffected by the global rounding mode.

Constant rounding modes (other than **FE\_DYNAMIC**) could be implemented on a platform using only dynamic rounding controls as illustrated in the following example:

```
{
        #pragma STDC FENV_ROUND direction
        // compiler inserts:
        // #pragma STDC FENV ACCESS ON
        // int savedrnd;
        // savedrnd = swapround(direction):
        ... operations and function calls affected by constant rounding mode ...
        // compiler inserts:
         // __savedrnd = __swapround(__savedrnd);
        ... function call that is not affected by constant rounding mode ...
        // compiler inserts:
        // __savedrnd = __swapround(__savedrnd);
        ... operations and function calls affected by constant rounding mode ...
        // compiler inserts:
        // __swapround(__savedrnd);
    }
where __swapround is defined as:
    static inline int __swapround(const int new) {
        const int old = fegetround();
        fesetround(new);
        return old:
    }
```

In F.8.1, change the second sentence from:

IEC 60559 requires that floating-point operations implicitly raise floating-point exception status flags, and that rounding control modes can be set explicitly to affect result values of floating-point operations. When the state for the **FENV\_ACCESS** pragma (defined in **<fenv.h>**) is "on", these changes to the floating-point state are treated as side effects which respect sequence points.364)

to:

IEC 60559 requires that floating-point operations implicitly raise floating-point exception status flags, and that rounding control modes can be set explicitly to affect result values of floating-point operations. These changes to the floating-point state are treated as side effects which respect sequence points.364)

Change footnote 364) from:

364) If the state for the **FENV\_ACCESS** pragma is "off", the implementation is free to assume the floating-point control modes will be the default ones and the floating-point status flags will not be tested, which allows certain optimizations (see F.9).

to:

364) If the state for the **FENV\_ACCESS** pragma is "off", the implementation is free to assume the dynamic floating-point control modes will be the default ones and the floating-point status flags will not be tested, which allows certain optimizations (see F.9).

In F.8.2, replace:

During translation the IEC 60559 default modes are in effect:

with:

During translation, constant rounding direction modes (7.6.2) are in effect where specified. Elsewhere, during translation the IEC 60559 default modes are in effect:

Change footnote 365) from:

365) As floating constants are converted to appropriate internal representations at translation time, their conversion is subject to default rounding modes and raises no execution-time floating-point exceptions (even where the state of the **FENV\_ACCESS** pragma is "on"). Library functions, for example **strtod**, provide execution-time conversion of numeric strings.

to:

365) As floating constants are converted to appropriate internal representations at translation time, their conversion is subject to constant or default rounding modes and raises no execution-time floating-point exceptions (even where the state of the **FENV\_ACCESS** pragma is "on"). Library functions, for example **strtod**, provide execution-time conversion of numeric strings.

In F.8.3, replace:

At program startup the floating-point environment is initialized ...

with:

At program startup the dynamic floating-point environment is initialized ...

In F.8.3, change the second bullet from:

The rounding direction mode is rounding to nearest.

to:

The dynamic rounding direction mode is rounding to nearest.

### 12 NaN support

IEC 60559 retains support for signaling NaNs. Although C11 notes that floating types may contain signaling NaNs, it does not otherwise specify signaling NaNs. Some unqualified references to NaNs in C11 do not properly apply to signaling NaNs, so that an implementation could not add signaling NaN support as an extension without contradicting C11. The goal of the following suggested changes is to allow implementations to conditionally support signaling NaNs as specified in IEC 60559, but to require only minimal support for signaling NaNs.

### Suggested changes to C11:

After 7.12 #5, add:

The signaling NaN macros

SNANF SNAN SNANL

each is defined if and only if the respective type contains signaling NaNs (5.2.4.2.2). They expand into a constant expression of the respective type representing a signaling NaN. If a signaling NaN macro is used as a static initializer for an object of the same type, the object is initialized with a signaling NaN value.

In 7.12.14, change 4th sentence from:

The following subclauses provide macros that are *quiet* (non floating-point exception raising) versions of the relational operators, and other comparison macros that facilitate writing efficient code that accounts for NaNs without suffering the "invalid" floating-point exception.

to:

Subclauses 7.12.14.1 through 7.12.14.6 provide macros that are quiet versions of the relational operators: the macros do not raise the "invalid" floating-point exception as an effect of quiet NaN arguments. The comparison macros facilitate writing efficient code that accounts for quiet NaNs without suffering the "invalid" floating-point exception.

In 7.12.14.1 through 7.12.14.5, append to "when  $\mathbf{x}$  and  $\mathbf{y}$  are unordered" the phrase "and neither is a signaling NaN"

In 7.12.14.6, append to the Description: "The unordered macro raises no floating-point exceptions if neither argument is a signaling NaN."

Change F.2.1 from:

### F.2.1 Infinities, signed zeros, and NaNs

This specification does not define the behavior of signaling NaNs.342) It generally uses the term *NaN* to denote quiet NaNs. The **NAN** and **INFINITY** macros and the **nan** functions in **<math.h>** provide designations for IEC 60559 NaNs and infinities.

to:

#### F.2.1 Infinities and NaNs

The **NAN** and **INFINITY** macros and the **nan** functions in **<math.h>** provide designations for IEC 60559 quiet NaNs and infinities. The **SNANF**, **SNAN**, and **SNANL** macros in **<math.h>** provide designations for IEC 60559 signaling NaNs.

This annex does not require the full support for signaling NaNs specified in IEC 60559. This annex uses the term *NaN*, unless explicitly qualified, to denote quiet NaNs. Where specification of signaling NaNs is not provided, the behavior of signaling NaNs is implementation defined (either treated as an IEC 60559 quiet NaN or treated as an IEC 60559 signaling NaN).

Any operator or **<math.h>** function that raises an "invalid" floating-point exception, if delivering a floating type result, shall return a quiet NaN.

In order to support signaling NaNs as specified in IEC 60559, an implementation should adhere to the following recommended practice.

## Recommended practice

Any floating-point operator or <math.h> function with a signaling NaN input, unless explicitly specified otherwise, raises an "invalid" floating-point exception.

NOTE Some functions do not propagate quiet NaN arguments. For example, hypot(x, y) returns infinity if x or y is infinite and the other is a quiet NaN. The recommended practice in this subclause specifies that such functions (and others) raise the "invalid" floating-point exception if an argument is a signaling NaN, which also implies they return a quiet NaN in these cases.

ISSUE: Should signaling NaN arguments cause domain errors? Should this be part of the above recommended practice?

The <fenv.h> header defines the macro

### FE\_SNANS\_ALWAYS\_SIGNAL

if and only if the implementation follows the recommended practice in this subclause.

Append to the end of F.5 the following paragraph:

The **fprintf** family of functions in **<stdio.h>** should behave as if floating-point operands were passed through **canonicalize()**.<sup>4</sup>

In F.9.2, bullet 1\*x and x/1 -> x, replace "are equivalent" with "may be regarded as equivalent".

In F.10 #3, change the last sentence:

The other functions in <math.h> treat infinities, NaNs, signed zeros, subnormals, and (provided the state of the FENV\_ACCESS pragma is "on") the floating-point status flags in a manner consistent with the basic arithmetic operations covered by IEC 60559.

to:

The other functions in <math.h> treat infinities, NaNs, signed zeros, subnormals, and (provided the state of the FENV\_ACCESS pragma is "on") the floating-point status flags in a manner consistent with IEC 60559 operations.

Append to footnote 374):

Note also that this implementation does not handle signaling NaNs as required of implementations that define FP\_SNANS\_ALWAYS\_SIGNAL.

Change footnotes 242) and 243) from:

- 242) NaN arguments are treated as missing data: if one argument is a NaN and the other numeric, then the **fmax** functions choose the numeric value. See F.10.9.2.
- 243) The **fmin** functions are analogous to the **fmax** functions in their treatment of NaNs.

to:

- 242) Quiet NaN arguments are treated as missing data: if one argument is a quiet NaN and the other numeric, then the **fmax** functions choose the numeric value. See F.10.9.2.
- 243) The **fmin** functions are analogous to the **fmax** functions in their treatment of quiet NaNs.

In G.3 #1, replace:

A complex or imaginary value with at least one infinite part is regarded as an *infinity* (even if its other part is a NaN).

with:

A complex or imaginary value with at least one infinite part is regarded as an *infinity* (even if its other part is a quiet NaN).

After G.6 #4, append the paragraph:

<sup>&</sup>lt;sup>4</sup> This is a recommendation instead of a requirement so that implementations may choose to print signaling NaNs differently from guiet NaNs.

In subsequent subclauses in G.6 "NaN" refers to a quiet NaN. The behavior of signaling NaNs in Annex G is implementation defined.

Change footnote 378) from:

378) As noted in G.3, a complex value with at least one infinite part is regarded as an infinity even if its other part is a NaN.

to:

378) As noted in G.3, a complex value with at least one infinite part is regarded as an infinity even if its other part is a quiet NaN.

## 13 Integer width macros

C11 clause 6.2.6.2 defines the *width* of integer types. These widths are needed in order to use the **fromfp, ufromfpx**, and **ufromfpx** functions to round to the integer types. The following suggested changes to C11 provide macros for the widths of integer types. On the belief that width macros would be generally useful, the proposal adds them to **limits.h>**.

Suggested changes to C11:

In 5.2.4.2.1, insert the following bullets, each after the current bullets for the same type:

- width of type charCHAR\_WIDTH 8
- width of type signed char
  - **SCHAR\_WIDTH 8**
- width of type unsigned char
  - **UCHAR\_WIDTH 8**
- width of type short int
  - SHRT\_WIDTH 16
- width of type unsigned short int
  - **USHRT\_WIDTH 16**
- width of type int
  - INT\_WIDTH 16
- width of type unsigned int
  - **UINT WIDTH 16**
- width of type long int
  - **LONG WIDTH 32**
- width of type unsigned long int
  - **ULONG\_WIDTH 32**
- width of type long long int
  - LLONG\_WIDTH 64
- width of type unsigned long long int
  - **ULLONG\_WIDTH 64**
- width of type intmax t
  - **INTMAX\_WIDTH 64**
- width of type uintmax\_t UINTMAX WIDTH 64

In 7.20.2.2, append

- width of minimum-width signed integer types
  - INT LEASTN WIDTH N
- width of minimum-width unsigned integer types

UINT\_LEASTN\_WIDTH A

In 7.20.2.3, append

- width of fastest minimum-width signed integer types
   INT\_FASTN\_WIDTH N
- width of fastest minimum-width unsigned integer types
   UINT\_FASTN\_WIDTH N

### 14 Mathematics <math.h>

The 2011 update to IEC 60559 requires several new operations that are appropriate for **<math.h>**. Also, in a few cases, it tightens requirements for functions that are already in C11 **<math.h>**.

### 14.1 Nearest integer functions

### 14.1.1 Round to integer value in floating type

IEC 60559 requires a function that rounds a value of floating type to an integer value in the same floating type, without raising the "inexact" floating-point exception, for each of the rounding methods: to nearest, to nearest even, upward, downward, and toward zero. The C11 **round**, **ceil**, **floor**, and **trunk** functions may meet this requirement for four of the five rounding methods, though are permitted to raise the "inexact" floating-point exception. The following suggested changes add a function that rounds to nearest and remove the latitude to raise the "inexact" floating-point exception.

### Suggested changes to C11:

Change F.10.6.1 #2:

The returned value is independent of the current rounding direction mode.

to:

The returned value is exact and is independent of the current rounding direction mode.

In F.10.6.1 #3, change:

result = rint(x); // or nearbyint instead of rint

to:

result = nearbyint(x);

Delete F.10.6.1 #4:

The **ceil** functions may, but are not required to, raise the "inexact" floating-point exception for finite non-integer arguments, as this implementation does.

Change F.10.6.2 #2:

The returned value is independent of the current rounding direction mode.

to:

The returned value is exact and is independent of the current rounding direction mode.

Delete the second sentence of F.10.6.2 #3: The **floor** functions may, but are not required to, raise the "inexact" floating-point exception for finite non-integer arguments, as that implementation does.

Change F.10.6.6 #2:

The returned value is independent of the current rounding direction mode.

to:

The returned value is exact and is independent of the current rounding direction mode.

Change F.10.6.6 #3 from:

The double version of round behaves as though implemented by

```
#include <math.h>
#include <fenv.h>
#pragma STDC FENV_ACCESS ON
double round(double x)
{
        double result;
        fenv_t save_env;
        feholdexcept(&save_env);
        result = rint(x);
        if (fetestexcept(FE_INEXACT)) {
                  fesetround(FE_TOWARDZERO);
                  result = rint(copysign(0.5 + fabs(x), x));
        }
        feupdateenv(&save_env);
        return result;
}
```

The **round** functions may, but are not required to, raise the "inexact" floating-point exception for finite non-integer numeric arguments, as this implementation does.

to:

The double version of round behaves as though implemented by

```
#include <math.h>
#include <fenv.h>
#pragma STDC FENV_ACCESS ON
double round(double x)
{
    double result;
    fenv_t save_env;
    feholdexcept(&save_env);
    result = rint(x);
    if (fetestexcept(FE_INEXACT)) {
        fesetround(FE TOWARDZERO);
        result = rint(copysign(0.5 + fabs(x), x));
        feclearexcept(FE_INEXACT);
    feupdateenv(&save env);
    return result:
}
```

After 7.12.9.6, add:

### 7.12.9.7 The roundeven functions

## **Synopsis**

#include <math.h>

```
double roundeven(double x);
float roundevenf(float x);
long double roundevenl(long double x);
```

### Description

The **roundeven** functions round their argument to the nearest integer value in floating-point format, rounding halfway cases to even (that is, to the nearest value whose least significant bit 0), regardless of the current rounding direction.

#### **Returns**

The roundeven functions return the rounded integer value.

After F.10.6.7, add:

#### F.10.6.8 The roundeven functions

- roundeven(±0) returns ±0.
- roundeven(±∞) returns ±∞.

The returned value is exact and is independent of the current rounding direction mode.

See the sample implementation for **ceil** in F.10.6.1.

In old F.10.6.8 #1, delete the second sentence: The returned value is exact.

Replace F.10.6.8 #2:

The returned value is independent of the current rounding direction mode. The **trunc** functions may, but are not required to, raise the "inexact" floating-point exception for finite non-integer arguments.

with:

The returned value is exact and is independent of the current rounding direction mode.

#### 14.1.2 Convert to integer type

IEC 60559 requires conversion operations from each of its formats to each integer format, signed and unsigned, for each of five different rounding methods. For each of these it requires an operation that raises the "inexact" floating-point exception (for non-integer in-range inputs) and an operation that does not raise the "inexact" floating-point exception. The suggested changes below satisfy this requirement with four new functions that take two extra arguments to represent the rounding direction and the rounding precision.

## Suggested changes to C11:

After 7.12 #6, add:

The math rounding direction macros

FP\_CEIL FP\_FLOOR FP\_TRUNC FP\_ROUND FP\_ROUNDEVEN

represent the rounding directions of the functions **ceil**, **floor**, **trunc**, **round**, and **roundeven**, respectively, that convert to integral values in floating-point formats. These macros are for use with the **fromfp**, **ufromfpx**, and **ufromfpx** functions.

After 7.12.9.8, add:

### 7.12.9.9 The fromfp and ufromfp functions

### **Synopsis**

```
#include <math.h>
intmax_t fromfp(double x, int round, unsigned int width);
intmax_t fromfpf(float x, int round, unsigned int width);
intmax_t fromfpl(long double x, int round, unsigned int width);
uintmax_t ufromfp(double x, int round, unsigned int width);
uintmax_t ufromfpf(float x, int round, unsigned int width);
uintmax_t ufromfpl(long double x, int round, unsigned int width);
```

### Description

The **fromfp** and **ufromfp** functions round **x**, using the math rounding direction indicated by **round**, to a signed or unsigned integer, respectively, of **width** bits, and return the result value in type **intmax\_t** or **uintmax\_t**, respectively. If the value of the **round** argument is not equal to the value of a math rounding direction macro, the direction of rounding is unspecified. If the **width** argument is 0, the functions return 0. If the value of **width** exceeds the width of the function type, the rounding is to the full width of the function type. The **fromfp** and **ufromfp** functions do not raise the "inexact" floating-point exception. If **x** is infinite or NaN or rounds to an integral value that is outside the range of integers of the specified width, the functions return an unspecified value and a domain error occurs.

#### Returns

The **fromfp** and **ufromfp** functions return the rounded integer value.

EXAMPLE Upward rounding of double  $\mathbf{x}$  to type  $\mathbf{int}$ , without raising the "inexact" floating-point exception, is achieved by

```
(int)fromfp(x, FP_CEIL, INT_WIDTH).
```

## 7.12.9.10 The fromfpx and ufromfpx functions

## **Synopsis**

```
#include <math.h>
intmax_t fromfpx(double x, int round, unsigned int width);
intmax_t fromfpxf(float x, int round, unsigned int width);
intmax_t fromfpxl(long double x, int round, unsigned int width);
uintmax_t ufromfpxf(double x, int round, unsigned int width);
uintmax_t ufromfpxf(float x, int round, unsigned int width);
uintmax_t ufromfpxl(long double x, int round, unsigned int width);
```

### **Description**

The **fromfpx** and **ufromfpx** functions differ from the **fromfp** and **ufromfp** functions, respectively, only in that the **fromfpx** and **ufromfpx** functions raise the "inexact" floating-point exception if a rounded result not exceeding the specified width differs in value from the argument **x**.

#### Returns

The **fromfpx** and **ufromfpx** functions return the rounded integer value.

NOTE Conversions to integer types that are not required to raise the inexact exception can be done simply by rounding to integral value in floating type and then converting to the target integer type. For example, the conversion of **long double x** to **uint64\_t**, using upward rounding, is done by

(uint64\_t)ceill(x)

After F.10.6.8, add:

#### F.10.6.9 The fromfp and ufromfp functions

The **fromfp** and **ufromfp** functions raise the "invalid" floating-point exception and return an unspecified value if the floating-point argument  $\mathbf{x}$  is infinite or NaN or rounds to an integral value that is outside the range of integers of the specified width.

These functions do not raise the "inexact" floating-point exception.

#### F.10.6.10 The fromfpx and ufromfpx functions

The **fromfpx** and **ufromfpx** functions raise the "invalid" floating-point exception and return an unspecified value if the floating-point argument  $\mathbf{x}$  is infinite or NaN or rounds to an integral value that is outside the range of integers of the specified width.

These functions raise the "inexact" floating-point exception if a valid result differs in value from the floating-point argument **x**.

## 14.2 The llogb functions

IEC 60559 requires that its logB operations, for invalid input, return a value outside  $\pm 2 \times (emax + p - 1)$ , where emax is the maximum exponent and p the precision of the floating-point input format. If the width of the int type is only 16 bits and the floating type has a 15-bit exponent (like the binary128 format), then the ilogb functions cannot meet this requirement. The following suggested changes to C11 add the llogb functions, which return long int and hence can satisfy this requirement for the long double types provided by current and expected implementations.

### Suggested changes to C11:

After 7.12 #8, add:

The macros

FP\_LLOGB0
FP\_LLOGBNAN

expand to integer constant expressions whose values are returned by <code>llogb(x)</code> if <code>x</code> is zero or NaN, respectively. The value of <code>FP\_LLOGB0</code> shall be either <code>LONG\_MIN</code> or <code>-LONG\_MAX</code>. The value of <code>FP\_LLOGBNAN</code> shall be either <code>LONG\_MIN</code>.

After 7.12.6.6, add:

### 7.12.6.7 The llogb functions

### **Synopsis**

#include <math.h>
long int llogb(double x);
long int llogbf(float x);
long int llogbl(long double x);

### **Description**

The **llogb** functions extract the exponent of x as a signed **long int** value. If x is zero they compute the value **FP\_LLOGB0**; if x is infinite they compute the value **LONG\_MAX**; if x is a NaN they compute the value **FP\_LLOGBNAN**; otherwise, they are equivalent to calling the corresponding **logb** function and casting the

returned value to type **long int**. A domain error or range error may occur if **x** is zero, infinite, or NaN. If the correct value is outside the range of the return type, the numeric result is unspecified.

#### Returns

The **llogb** functions return the exponent of **x** as a signed **long int** value.

Forward references: the logb functions (7.12.6.?).

ISSUE: Should the 7.12 specification of llogb require a domain error for finite out-of-range cases? The C committee didn't want this for ilogb.

After F.10.3.5, add:

### F.10.3.6 The llogb functions

The **llogb** functions are equivalent to the **ilogb** functions, except that the **llogb** functions determine a result in the **long int** type.

### 14.3 Max-min magnitude functions

IEC 60559 requires functions that determine which of two inputs has the maximum and minimum magnitude.

#### Suggested changes to C11:

After 7.12.12.3, add:

#### 7.12.12.3 The fmaxmag functions

#### **Synopsis**

```
#include <math.h>
double fmaxmag(double x, double y);
float fmaxmagf(float x, float y);
long double fmaxmagl(long double x, long double y);
```

## Description

The **fmaxmag** functions determine the numeric value of their argument whose magnitude is the maximum of the magnitudes of the arguments.<sup>5</sup>

## Returns

The fmaxmag functions return the numeric value of their argument of maximum magnitude.

### 7.12.12.3 The fminmag functions

#### **Synopsis**

```
#include <math.h>
double fminmag(double x, double y);
float fminmagf(float x, float y);
long double fminmagl(long double x, long double y);
```

### Description

\_

<sup>&</sup>lt;sup>5</sup> Quiet NaN arguments are treated as missing data: if one argument is a quiet NaN and the other numeric, then the **fmaxmag** functions choose the numeric value. See F.10.9.4.

The **fminmag** functions determine the numeric value of their argument whose magnitude is the minimum of the magnitudes of the arguments.<sup>6</sup>

#### Returns

The **fminmag** functions return the numeric value of their argument of minimum magnitude.

After F.10.9.3, add:

#### F.10.9.4 The fmaxmag functions

If just one argument is a NaN, the **fmaxmag** functions return the other argument (if both arguments are NaNs, the functions return a NaN).

The returned value is exact and is independent of the current rounding direction mode.

The body of the **fmaxmag** function might be<sup>7</sup>

```
{ return (isgreaterequal(fabs(x), fabs(y)) || isnan(y)) ? x : y; }
```

### F.10.9.5 The fminmag functions

The **fminmag** functions are analogous to the **fmaxmag** functions (F.10.9.4).

The returned value is exact and is independent of the current rounding direction mode.

### 14.4 The nextup and nextdown functions

IEC 60559 replaces the previously recommended two-argument nextAfter operation with one-argument nextUp and nextDown operations. C11 supports the nextAfter operation with the **nextafter** and **nexttoward** functions. The following suggested changes to C11 add functions for the new operations and retain the **nextafter** and **nexttoward** functions already in C11.

### Suggested changes to C11:

After 7.12.11.2 add:

### 7.12.11.3 The nextup functions

### **Synopsis**

```
#include <math.h>
double nextup(double x);
float nextupf(float x);
long double nextupl(long double x);
```

### Description

The **nextup** functions determine the next representable value, in the type of the function, greater than  $\mathbf{x}$ . If  $\mathbf{x}$  is the negative number of least magnitude in the type of  $\mathbf{x}$ ,  $\mathbf{nextup}(\mathbf{x})$  is -0 if the type has signed zeros and is 0 otherwise. If  $\mathbf{x}$  is zero,  $\mathbf{nextup}(\mathbf{x})$  is the positive number of least magnitude in the type of  $\mathbf{x}$ .  $\mathbf{nextup}(\mathbf{HUGE\_VAL})$  is  $\mathbf{HUGE\_VAL}$ .

### Returns

<sup>&</sup>lt;sup>6</sup> The **fminmag** functions are analogous to the **fmaxmag** functions in their treatment of quiet NaNs.

<sup>&</sup>lt;sup>7</sup> This implementation does not handle signaling NaNs as required of implementations that define FP\_SNANS\_ALWAYS\_SIGNAL.

The **nextup** functions return the next representable value in the specified type greater than **x**.

#### 7.12.11.4 The nextdown functions

### **Synopsis**

```
#include <math.h>
double nextdown(double x);
float nextdownf(float x);
long double nextdownf(long double x);
```

#### **Description**

The **nextdown** functions determine the next representable value, in the type of the function, less than  $\mathbf{x}$ . If  $\mathbf{x}$  is the positive number of least magnitude in the type of  $\mathbf{x}$ ,  $\mathbf{nextdown}(\mathbf{x})$  is +0 if the type has signed zeros and is 0 otherwise. If  $\mathbf{x}$  is zero,  $\mathbf{nextdown}(\mathbf{x})$  is the negative number of least magnitude in the type of  $\mathbf{x}$ .  $\mathbf{nextdown}(\mathbf{HUGE\_VAL})$  is - $\mathbf{HUGE\_VAL}$ .

#### **Returns**

The **nextdown** functions return the next representable value in the specified type less than **x**.

After F.10.8.2, add:

### F.10.8.3 The nextup functions

- nextup(+∞) returns +∞.
- nextup(-∞) returns the largest-magnitude negative finite number in the type of the function.

#### F.10.8.4 The nextdown functions

- nextdown(+\infty) returns the largest-magnitude positive finite number in the type of the function.
- nextdown(-∞) returns -∞.

## 14.5 Functions that round result to narrower type

IEC 60559 requires add, subtract, multiply, divide, fused multiply-add, and square root operations that round once to a floating-point format independent of the format of the operands. The following suggested changes to C11 add functions for these operations that round to formats narrower than the operand formats. The operations that round to the same and wider formats are already available by casting operands of the built-in operators (+, -, \*, /) to the desired type and by calling the **fma** and **sqrt** functions of the desired type.

### Suggested changes to C11:

After 7.12.13, add:

### 7.12.14 Functions that round result to narrower type

### 7.12.14.1 Add and round to narrower type

## **Synopsis**

```
#include <math.h>
float fadd(double x, double y);
float faddl(long double x, long double y);
double daddl(long double x, long double y);
```

## Description

These functions compute the sum  $\mathbf{x} + \mathbf{y}$ , rounded to the type of the function. They compute the sum (as if) to infinite precision and round once to the result format, according to the current rounding mode. A range error may occur for finite arguments. A domain error may occur for infinite arguments.

#### Returns

These functions return the sum x + y, rounded to the type of the function.

#### 7.12.14.2 Subtract and round to narrower type

### **Synopsis**

```
#include <math.h>
float fsub(double x, double y);
float fsubl(long double x, long double y);
double dsubl(long double x, long double y);
```

### Description

These functions compute the difference  $\mathbf{x}$  -  $\mathbf{y}$ , rounded to the type of the function. They compute the difference (as if) to infinite precision and round once to the result format, according to the current rounding mode. A range error may occur for finite arguments. A domain error may occur for infinite arguments.

#### **Returns**

These functions return the difference x - y, rounded to the type of the function.

### 7.12.14.3 Multiply and round to narrower type

### **Synopsis**

```
#include <math.h>
float fmul(double x, double y);
float fmull(long double x, long double y);
double dmull(long double x, long double y);
```

#### **Description**

These functions compute the product  $\mathbf{x} \times \mathbf{y}$ , rounded to the type of the function. They compute the product (as if) to infinite precision and round once to the result format, according to the current rounding mode. A range error may occur for finite arguments. A domain error occurs for one infinite argument and one zero argument.

#### Returns

These functions return the product of  $\mathbf{x} \times \mathbf{y}$ , rounded to the type of the function.

### 7.12.14.4 Divide and round to narrower type

## **Synopsis**

```
#include <math.h>
float fdiv(double x, double y);
float fdivl(long double x, long double y);
double ddivl(long double x, long double y);
```

#### Description

These functions compute the quotient  $\mathbf{x} \div \mathbf{y}$ , rounded to the type of the function. They compute the quotient (as if) to infinite precision and round once to the result format, according to the current rounding mode. A

range error may occur for finite arguments. A domain error occurs for either both arguments infinite or both arguments zero. A pole error occurs for a finite x and a zero y.

#### **Returns**

These functions return the quotient  $\mathbf{x} \div \mathbf{y}$ , rounded to the type of the function.

#### 7.12.14.5 Floating multiply-add rounded to narrower type

#### **Synopsis**

```
#include <math.h>
float ffma(double x, double y, double z);
float ffmal(long double x, long double y, long double z);
double dfmal(long double x, long double y, long double z);
```

### **Description**

These functions compute  $(\mathbf{x} \times \mathbf{y}) + \mathbf{z}$ , rounded to the type of the function. They compute  $(\mathbf{x} \times \mathbf{y}) + \mathbf{z}$  to infinite precision and round once to the result format, according to the current rounding mode. A range error may occur for finite arguments. A domain error may occur for an infinite argument.

#### Returns

These functions return  $(\mathbf{x} \times \mathbf{y}) + \mathbf{z}$ , rounded to the type of the function.

### 7.12.14.6 Square root rounded to narrower type

### **Synopsis**

```
#include <math.h>
float fsqrt(double x);
float fsqrtl(long double x);
double dsqrtl(long double x);
```

### **Description**

These functions compute the square root of  $\mathbf{x}$ , rounded to the type of the function. They compute the square root (as if) to infinite precision and round once to the result format, according to the current rounding mode. A range error may occur for finite positive arguments. A domain error occurs for negative arguments.

### Returns

These functions return the square root of  $\mathbf{x}$ , rounded to the type of the function.

After old F.10.10 add:

#### F.10.11 Functions that round result to narrower type

The functions that round their result to narrower type (7.12.14) are fully specified in IEC 60559. The returned value is dependent on the current rounding direction mode.

#### 14.6 Comparison macros

IEC 60559 requires an extensive set of comparison operations. C11's built-in equality and relational operators and quiet comparison macros and their negations (!) support all these required operations, except for compareSignalingEqual and compareSignalingNotEqual. The following suggested changes to C11 provide a function-like macro for compareSignalingEqual. The negation of the macro provides compareSignalingNotEqual. See Table 1.

### Suggested changes to C11:

After 7.12.14.6, add:

#### 7.12.14.7 The iseqsig macro

### **Synopsis**

```
#include <math.h>
int iseqsig(floating-type x, floating-type y);
```

### **Description**

The **iseqsig** macro determines whether its arguments are equal. A domain error occurs if an argument is a NaN.

#### Returns

The iseqsig macro returns 1 if its arguments are equal and 0 otherwise.

After F.10.11, add:

## F.10.11.1 The iseqsig macro

The equality operator == and the **iseqsiq** macro produce equivalent results, except that the **iseqsig** macro raises the "invalid" floating-point exception if an argument is a NaN.

## 14.7 Inquiry macros

IEC 60559 requires several inquiry operations, all but three of which are already supported in C11 as function-like macros. The suggested changes to C11 below support the remaining three.

### Suggested change to C11:

In 7.12.3, add:

### 7.12.3.? The issubnormal macro

#### **Synopsis**

```
#include <math.h>
int issubnormal(real-floating x);
```

### **Description**

The **issubnormal** macro determines whether its argument value is subnormal. First, an argument represented in a format wider than its semantic type is converted to its semantic type. Then determination is based on the type of the argument.

#### Returns

The issubnormal macro returns a nonzero value if and only if its argument is subnormal.

### 7.12.3.? The issignaling macro

#### **Synopsis**

#include <math.h>

## int issignaling(real-floating x);

### Description

The **issignaling** macro determines whether its argument value is a signaling NaN, without raising a floating-point exception.

#### **Returns**

The issignaling macro returns a nonzero value if and only if its argument is a signaling NaN.

### 7.12.3.? The iscanonical macro

### **Synopsis**

```
#include <math.h>
int iscanonical(real-floating x);
```

### Description

The **iscanonical** macro determines whether its argument value is canonical (5.2.4.2.2). First, an argument represented in a format wider than its semantic type is converted to its semantic type. Then determination is based on the type of the argument.

#### **Returns**

The **iscanonical** macro returns a nonzero value if and only if its argument is canonical.

### 14.8 Total order functions

IEC 60559 requires a totalOrder operation, which it defines as follows:

"totalOrder(x, y) imposes a total ordering on canonical members of the format of x and y:

- a) If x < y, totalOrder(x, y) is true.
- b) If x > y, totalOrder(x, y) is false.
- c) If x = y:
  - 1) totalOrder(-0, +0) is true.
  - 2) totalOrder(+0, -0) is false.
  - 3) If *x* and *y* represent the same floating-point datum:
    - i) If x and y have negative sign, totalOrder(x, y) is true if and only if the exponent of  $x \ge$  the exponent of y
    - ii) otherwise totalOrder(x, y) is true if and only if the exponent of  $x \le$  the exponent of y.
- d) If x and y are unordered numerically because x or y is NaN:
  - 1) totalOrder(-NaN, y) is true where -NaN represents a NaN with negative sign bit and y is a floating-point number.
  - 2) totalOrder(x, +NaN) is true where +NaN represents a NaN with positive sign bit and x is a floating-point number.
  - 3) If x and y are both NaNs, then totalOrder reflects a total ordering based on:
    - i) negative sign orders below positive sign
    - ii) signaling orders below quiet for +NaN, reverse for -NaN
    - iii) lesser payload, when regarded as an integer, orders below greater payload for +NaN, reverse for -NaN."

IEC 60559:2011 also requires a totalOrderMag operation which is the totalOrder of the absolute values of the operands. The following suggested change to C11 provides these operations.

### Suggested change to C11:

After F.10.11, add:

#### F.10.12 The total order functions

This annex specifies the total order functions required IEC 60559.8

#### F.10.12.1 The totalorder functions

### **Synopsis**

```
#include <math.h>
int totalorder(double x, double y);
int totalorderf(float x, float y);
int totalorderl(long double x, long double y);
```

#### **Description**

The **totalorder** functions determine whether the total order relationship, defined by IEC 60559, is true for the ordered pair of its arguments  $\mathbf{x}$ ,  $\mathbf{y}$ . The functions are fully specified in IEC 60559.

#### Returns

The **totalorder** functions return nonzero if and only if the total order relation is true for the ordered pair of its arguments  $\mathbf{x}$ ,  $\mathbf{y}$ .

#### F.10.12.2 The totalordermag functions

### **Synopsis**

```
int totalordermag(double x, double y);
int totalordermagf(float x, float y);
int totalordermagl(long double x, long double y);
```

## Description

The **totalordermag** functions determine whether the total order relationship, defined by IEC 60559, is true for the ordered pair of the magnitudes of its arguments  $\mathbf{x}$ ,  $\mathbf{y}$ . The functions are fully specified in IEC 60559.

### Returns

The **totalordermag** functions return nonzero if and only if the total order relation is true for the ordered pair of the magnitudes of it arguments **x**, **y**.

### 14.9 The canonicalize functions

IEC 60559 requires an arithmetic convertFormat operation from each format to itself. This operation produces a canonical encoding and, for a signaling NaN input, raises the "invalid" floating-point and delivers a quiet NaN. C assignment (and conversion as if by assignment) to the same format may be implemented as a convertFormat operation or as a copy operation. The suggested change to C11 below provides the IEC 60559 convertFormat operation.

#### Suggested change to C11:

After new F.10.8.4, add:

 $<sup>^{8}</sup>$  The total order functions are specified only in Annex F because they depend on the details of IEC 60559 formats.

#### F.10.8.5 The canonicalize functions

### **Synopsis**

```
#include <math.h>
double canonicalize(double x);
float canonicalizef(float x);
long double canonicalizel(long double x);
```

#### Description

The **canonicalize** functions produce  $^9$  the canonical version of the argument. For a signaling NaN argument, the "invalid" floating-point exception is raised and a quiet NaN (which should be that signaling NaN made quiet) is produced. For quiet NaN, infinity, and finite arguments, the functions raise no floating-point exceptions and return canonical  $\mathbf{x}$ .

#### Returns

The functions return a canonical result.

ISSUE: Is Annex F the right place for canonicalize? It's awkward in 7.12 because implementations may have values with no equivalent canonical value. We could have

int canonicalize(double \* cx, const double \* x);

returning non-zero if a canonical encoding equal to x is not provided.

## 14.10 NaN payload functions

IEC 60559 defines the payload of a NaN to be a certain part of the NaN's significand interpreted as an integer. The payload is intended to provide implementation-defined diagnostic information about the NaN, such as where or how the NaN was created. The following suggested changes to C11 provide functions to get and set the NaN payloads defined in IEC 60559.

## Suggested change to C11:

After new F.10.12, add:

#### F.10.13 Payload functions

#### F.10.13.1 The getpayload functions

#### **Synopsis**

```
#include <math.h>
double getpayload(const double *x );
float getpayloadf(const double *x );
long double getpayloadl(const double *x );
```

### Description

The **getpayload** functions extract the payload (treated as an integer) of a NaN input and returns that integer as a floating-point value. The sign of the returned integer is positive. Floating-point exceptions are not raised for signaling NaN arguments. If \*x is not a NaN, the return result is unspecified.

### **Returns**

29

<sup>&</sup>lt;sup>9</sup> As if **x\*1e0** were computed.

The functions return a floating-point representation of the integer value of the payload of the NaN input.

### F.10.13.2 The setpayload functions

### **Synopsis**

```
#include <math.h>
int setpayload(double *res, double pl);
int setpayloadf(float *res, double pl);
int setpayloadl(long double *res, double pl);
```

### Description

The **setpayload** functions create a quiet NaN with the payload specified by **pl** and a zero sign bit and stores that NaN into the object pointed to by **\*res**. If **pl** is not a positive floating-point integer representing a valid payload, **\*res** is set to positive zero.

### **Returns**

If the functions stored the specified NaN, the functions return a zero value, otherwise a non-zero value (and \*res is set to zero).

## F.10.13.3 The setpayloadsig functions

### **Synopsis**

```
#include <math.h>
int setpayloadsig(double *res, double pl);
int setpayloadsigf(float *res, double pl);
int setpayloadsigl(long double *res, double pl);
```

### **Description**

The **setpayloadsig** functions create a signaling NaN with the payload specified by **pl** and a zero sign bit and stores that NaN into the object pointed to by **\*res**. If **pl** is not a positive floating-point integer representing a valid payload, **\*res** is set to positive zero.

#### **Returns**

If the functions stored the specified NaN, the functions return a zero value, otherwise a non-zero value (and \*res is set to zero).

## 15 The floating-point environment <fenv.h>

## 15.1 The fesetexcept function

IEC 60559 requires a raiseFlags operation that sets floating-point exception flags. Unlike the C **feraiseexcept** function in **<fenv.h>**, the raiseFlags operation does not cause side effects (notably traps) as could occur if the exceptions resulted from arithmetic operations. The following suggested change to C11 provides the raiseFlags operation.

## Suggested change to C11:

After 7.6.2.3, add:

### 7.6.2.4 The fesetexcept function

### **Synopsis**

#include <fenv.h>
int fesetexcept(int excepts);

#### Description

The **fesetexcept** function attempts to set the supported floating-point exception flags represented by its argument. This function does not clear any floating-point exception flags. This function changes the state of the floating-point exception flags, but does not cause any other side effects that might be associated with raising floating-point exceptions.<sup>10</sup>

#### Returns

The **fesetexcept** functions returns zero if all the specified exceptions were successfully set or if the **excepts** argument is zero or. Otherwise, it returns a nonzero value.

### 15.2 The fetestexceptflag function

IEC 60559 requires a testSavedFlags operation to test saved representations of floating-point exception flags. This differs from the C **fetestexcept** function in **<fenv.h>** which tests floating-point exception flags directly. The following suggested change to C11 provides the testSavedFlags operation.

### Suggested change to C11:

After old 7.6.2.4, add:

#### 7.6.2.5 The fetestexceptflag function

### **Synopsis**

```
#include <fenv.h>
int fetestexceptflag(const fexcept_t * flagp, int excepts);
```

### **Description**

The **fetestexceptflag** determines which of a specified subset of the floating-point exception flags are set in the object pointed to by **flagp**. The value of **\*flagp** shall have been set by a previous call to **fegetexceptflag**. The **excepts** argument specifies the floating-point status flags to be queried.

### Returns

The **fetestexcept** function returns the value of the bitwise OR of the floating-point exception macros included in **excepts** corresponding to the floating-point exceptions set in **\*flagp.** 

### 15.3 Control modes

IEC 60559 requires a saveModes operation that saves all the user-specifiable dynamic floating-point modes supported by the implementation, including dynamic rounding direction and trap enablement modes. The following suggested changes to C11 support this operation.

### Suggested changes to C11:

After 7.6 #5, add:

The type

femode\_t

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<sup>&</sup>lt;sup>10</sup> Enabled traps for floating-point exceptions are not taken.

represents the collection of dynamic floating-point control modes supported by the implementation, including the dynamic rounding direction mode.

After old 7.6 #8, add:

The macro

### FE\_DFL\_MODE

represents the default state for the collection of dynamic floating-point control modes supported by the implementation - and has type "pointer to const-qualified **femode\_t**".

Additional implementation-defined states for the dynamic mode collection, with macro definitions beginning with **FE**\_ and an uppercase letter, and having type "pointer to const-qualified **femode\_t**", may also be specified by the implementation.

Rename 7.6.3:

### 7.6.3 Rounding and other control modes

Append to 7.6.3 #1:

The **fegetmode** and **fesetmode** functions manage all the implementation's dynamic floating-point control modes collectively.

After 7.6.3 #1. insert:

### 7.6.3.1 The fegetmode function

## **Synopsis**

```
#include <fenv.h>
int fegetmode(femode_t *modep);
```

#### **Description**

The **fegetmode** function attempts to store all the dynamic floating-point control modes into the object pointed to by **modep**.

#### Returns

The **fegetmode** function returns zero if the modes were successfully stored. Otherwise, it returns a nonzero value.

After old 7.6.3.1, add:

### 7.6.3.3 The fesetmode function

## **Synopsis**

```
#include <fenv.h>
int fesetmode(const fenv_t *modep);
```

### Description

The **fesetmode** function attempts to establish the dynamic floating-point modes represented by the object pointed to by **modep**. The argument **modep** shall point to an object set by a call to **fegetmode**, or equal **FE\_DFLT\_MODE** or a dynamic floating-point mode state macro defined by the implementation.

## Returns

The **fesetmode** function returns zero if the modes were successfully established. Otherwise, it returns a nonzero value.

# **Bibliography**

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- [4] IEC 60559:1989, Binary floating-point arithmetic for microprocessor systems, second edition
- [5] IEEE 754-2008, IEEE Standard for Floating-Point Arithmetic
- [6] IEEE 754-1985, IEEE Standard for Binary Floating-Point Arithmetic
- [7] IEEE 854-1987, IEEE Standard for Radix-Independent Floating-Point Arithmetic